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Sea level muon spectrum below I TeV range derived from the latest primary spectrum

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Abstract : Muon energy spectrum in the vertical direction has been estimated from the recent primary cosmic ray nucleon spectrum expected from the direct measurements We have found that the fit to the data of different primary elemental groups *viz* p, He, CNO, Ne-S and Fe favoured different spectral indices These directly measured primary mass composition results are precise and all particle primary nucleon spectrum has been constructed using superposition model for the calculation of the energy spectra of the cosmic ray secondaries in the atmosphere with the adoption of Feynman scaling hypothesis.

Using such primary nucleon spectrum and the Z-factors from the CERN LEBC-EHS data on the Lorentz invariant cross section results on $pp \rightarrow \pi^{\pm}X$ and $pp \rightarrow K^{\pm}X$ inclusive reactions and FNAL data on $\pi^{\pm}p \rightarrow \pi^{\pm}X$ reactions for the estimation of hadronic energy moments duly corrected for A-A collisions the meson energy spectrum in the atmosphere have been calculated. The conventional meson atmospheric diffusion equation is employed here for the estimation of vertical muon energy spectrum from the conventional non-prompt meson decays.

The derived differential sea level muon energy spectrum for energies $\leq$ 1 TeV has been found to follow the power law fit of the form $N_{\mu}(E) = 0.55E^{-3.16}$ Our estimated vertical muon energy spectrum has been compared with the global muon flux data We have also estimated the contribution of pions and kaons separately to the total muon flux and have found that ratio of the muon spectrum estimated from kaon decay to the muon spectrum from the pion decay increases with energy.

Keywords : Primary cosmic rays, ground level muon energy spectrum, observed results

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1. Introduction

Investigation on high energy cosmic rays is necessary to understand their energy spectrum, mass composition and to confirm their arrival direction from astronomical sources.

In general the earth's atmosphere is opaque to conventional cosmic rays like protons, nuclei, electrons and photons but the directly measured particle spectra are available at the top of the atmosphere, those have been detected by balloon and satellite borne active and passive

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detector experiments. So the atmosphere may be treated as target for the study of astroparticle interactions with low Z nuclei like nitrogen and oxygen nuclei present in the atmosphere to estimate the spectral characteristics of the secondary components produced through the extensive air showers. Among the shower components high energy muons are unique which can give some idea about the hadronic stage of the nucleon cascade [1-4]. In general secondary cosmic-ray spectra in the atmosphere are dependent on the primary elemental abundance. The Fuji Kanbala observation [5] shows the absence of knee of the primary proton spectrum beyond 1000 TeV energy. In a recent investigation Mitra *et al.* [6] have surveyed the elemental spectra of primary cosmic rays obtained from JACEE [7], MSU [8], SOKOL [9] and CRN [10] and pointed out that the elemental spectra exhibit different energy spectral indices and can be extrapolated for the derivation of secondary muon energy spectra at super high energies. Swordy [11] has exhibited that below 1 PeV energy the elemental spectral shapes differ from one another with the EAS predictions of Maryland and MIT experiments [12]. Due to the scarcity of high energy A-A collision accelerator data and limitation of the nucleon cascade formulation till date the derivation of precise spectra of the secondary components emitted by primary cosmic rays is an interesting topic of investigation.

Using the primary nucleon spectrum obtained from the directly measured data [7-10, 13-16] we have derived the vertical muon energy spectrum by adopting the formulation [17] for energies $\leq$ 1 TeV generated from the decay of non-prompt mesons which are the initial air shower interaction products initiated by the primary nucleon air collisions. The CERN LEBC-EHS [18] data for pp collisions and FNAL [19] data for πp collisions have been considered for hadronic energy moments estimation. Pion production by secondary pions have been taken into account and the final total muon spectrum from the $pp \rightarrow \pi^{\pm}X$, $pp \rightarrow K^{\pm}X$ and $\pi^{\pm}p \rightarrow \pi^{\pm}X$ decays has been compared with the vertical muon spectrum results obtained from the global survey of Pal and Bhattacharyya [20].

2. Nuclear physics

The elemental i-th species energy spectra of the primary cosmic rays can be represented by the power law fit of the form

$$n_i(E)\, dE = K_i E^{-(\gamma_i+1)} dE, \tag{1}$$

the terms K_i and γ_i are the spectral amplitudes and integral spectral indices γ_i of the i-th species, respectively. The conventional superposition model [21, 22] converts the primary nuclei spectrum to the nucleon spectrum by the form

$$N_i(E)\, dE = \sum_{i=H}^{Fe} A_i K_i E^{-(\gamma_i+1)} dE = KE^{-(\gamma+1)} dE, \tag{2}$$

where A_i represents the mass number of the i-th elemental species.

The accelerator data on the Lorentz invariant cross-section for the inclusive reactions $pp \rightarrow \pi^{\pm}X$, $pp \rightarrow K^{\pm}X$ and $\pi p \rightarrow \pi^{\pm}X$ as functions of the Feynman variable x by its p_T integrated invariant form

$$x\left(\frac{d\sigma}{dx}\right) = C(1-x)^n, \tag{3}$$

where C and n are the fitting parameters.

The spectrum weighted moments for hadrons in air for the inclusive reactions $a + b \rightarrow c + X$ can be calculated by using the form

$$Z_{ac} = \frac{C\Gamma(\gamma)\,\Gamma(n+1)}{\sigma_{in}\Gamma(\gamma+n+1)}, \tag{4}$$

where σ_{in} is the inelastic interaction cross-section of the reacting system. Such Z-factors are to be corrected for pA and AA collisions by following the procedure of Minorikawa and Mitsui [23]. The production spectra of i-mesons in the atmosphere follows the relation :

$$i(E)\,dE = \left(Z_{pi^+} + Z_{pi} \right) N_i(E)\,dE. \tag{5}$$

The kinetic equation for the passage of $i\,(\pi^{\pm}, K^{\pm})$ in air at the atmospheric depth y follows

$$\frac{\delta i}{\delta E} = \frac{i(E)\,exp[-y/\Lambda_p]}{\lambda_p} - (1/\lambda_i + H_i/[yE])\,i(E,y), \tag{6}$$

where λ_p and λ_i are the interaction mean free path of nucleons and i-mesons in the atmosphere, respectively ; H_i is the critical energy of i-meson decay ; Λ_p is the absorption mean free path of nucleons in air. The solution to the above stated differential equation for i-mesons generated through $pp \rightarrow i^{\pm}X$ inclusive reaction channels considered for muon flux estimation from N-generations of the parent mesons by using the procedure used earlier by Bhattacharyya and Pal [17] :

$$M_{pi}(E)\,dE = \alpha^{(N-1)\gamma} D^N_{pi}(E)dE, \tag{7}$$

where α is the elasticity, and

$$D^N_{pi}(E) = N_i(E) Z'^{\,N}_{pi} A_i(\lambda_i/\lambda_p)^N \frac{1}{(N-1)!};$$

$$\sum_{m=1}^{10} \frac{(1-\lambda_i/\Lambda_p)^{m-1} m(m+1)\ldots(m+N-1)}{1+(m+N-1)R_i E/H_i(\theta)}. \tag{8}$$

$N_i(E)$ represents the primary elemental spectrum ;

$$A_i = \frac{1-r_i^{2(\gamma+1)}}{(\gamma+1)(1-r_i^2)}; \quad R_i = \frac{(\gamma+2)}{(\gamma+1)} \frac{1-r_i^{2(\gamma+1)}}{1-r_i^{2(\gamma+2)}}$$

$r_i = (m_i^2 + m_\mu^2)/(2m_i^2)$; $H_i = m_i c^2 H/(c\tau_i)$; H is the scale height of the atmosphere ; m_i and τ_i are the mass and life time of i-mesons, *viz.* for $i = \pi^{\pm}$ or $K^{\pm}$, respectively.

In a similar way [3], the muon flux at an atmospheric depth y emitted from the first generation of pions for N = 1 produced from the inelastic collisions with the atmospheric nuclei can be estimated by using the following relation

$$M^1_{\pi\pi}(E,y) = \frac{\lambda_\pi A_\pi Z^{1A}_{\pi\pi} N(E)}{\lambda_p[1+2R_\pi E/H_\pi(\theta)}\left[1 - b_{\pi\pi}(H_\pi(\theta)/r_\pi E)^{a_{\pi\pi}}\right]. \tag{9}$$

The total diffuse muon energy spectrum at a final atmospheric depth y_0g – cm^{-2} emitted from the decay of non-prompt mesons using the form

$$M_\mu^{atm}(E, y_0)dE = \sum_{N=1}^{10} M_{p\pi}^{N}(E, y_0)dE$$

$$+\sum_{N=1}^{10} b_{K_\mu} M_{pK}^{N}(E, y_0)dE + M_{\pi\pi}^{1}(E, y_0)dE \quad W(E, y, y_0), \tag{10}$$

where $W(E, y, y_0)$ represents the survival probability of muons at an atmospheric depth y g-cm^{-2} that should be observed at a depth y_0 g-cm^{-2} with muon energy E whose simplified form follows

$$W(E, y, y_0) = \left[\frac{y}{y_0}\frac{E}{E+(\alpha+\beta E)(y_0-y)}\right]^{H_\mu/[E+(\alpha+\beta E)y_0]} \tag{11}$$

where α and β are the energy loss parameters ; first one for ionisation and the other due to the pair production, bremmstrahlung and nuclear interactions. The standard energy loss formulation for muons follows the form

$$-\frac{dE}{dy} = \alpha + \beta E. \tag{12}$$

3. Results and discussion

The primary elemental energy spectra available from direct measurements by using balloon and satellite borne active and passive detectors by different groups [7-10, 13-16] have been fitted by the power law fits of the form (1) and their fitting spectral amplitudes K_i (cm^2.s.sr.GeV/n)$^{-1}$ and integral spectral indices γ_i are displayed in the Table 1.

Table 1. Calculated values of the spectral amplitudes and indices

Elements	A_i	K_i (cm^2.s sr GeV/n)$^{-1}$	γ_i
H	1	12216 9 × 10^{-4}	1.68
He	4	423 97 × 10^{-4}	1 59
CNO	14	26 004 × 10^{-4}	1 57
$Ne - Si$	28	6 6350 × 10^{-4}	1.57
Fe	56	1.5477 × 10^{-4}	1.53

Table 1 shows the calculated values of the spectral amplitudes K_i (cm^2.s.sr.GeV/n)$^{-1}$ and indices γ_i obtained from the statistical fits to the elemental primary flux data [7-10, 13-16].

Using the relation (2) and the parameters from the Table 1, the all nucleon spectrum at the top of the atmosphere has been estimated and it follows the form :

$$N(E)dE = 1.42E^{-2.66}dE\ [\text{cm}^2.\text{s.sr.GeV}/\text{n}]^{-1}. \tag{13}$$

The derived all nucleon spectrum has been compared in the Figure 1 with the similar results found by earlier authors presented by [24-29].

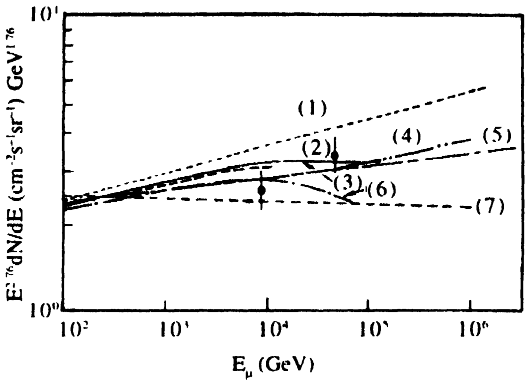

Figure 1. The derived all nucleon spectrum at the top of the atmosphere is represented by the full curve and has been compared with the earlier results Curves represent the results, 1-Volkova *et al* [24] , 2-Present Work , 3-Allkofer *et al* [25] , 4-Honda *et al* [26] , 5-Mitsui *et al* [27] , 6-Agrawal *et al* [28] , 7-Butkevich *et al* [29] Experimental data , ● JACEE data [7]

The plot indicates that the present result is well in accord with the earlier result expected from the DEIS magnetic spectrograph muon flux result [24]. The extrapolated result is supported by JACEE data [7].

Using the relation (2)-(4) along with the spectrum (with integral spectral index $\gamma = 1.66$) weighted moments Z_{hi} for inclusive reactions $pp \to i^{\pm} X$ have been calculated from the CERN [18] and FNAL [19] data are duly corrected by adopting the procedure of Minorikawa and Mitsui [23] and are displayed in the Table 2.

Table 2. Parametric values of the fitting constants C and n of the relation (3)

Projectile	meson	C	n	Z_{ph}	Ref
p	π^+	22 8	3 08	$Z_{p\pi^+}$ = 0 05014	CERN data [18]
p	π^-	28 89	5.35	$Z_{p\pi^-}$ = 0 03189	CERN data [18]
p	K^+	2 79	4.34	Z_{pK^+} = 0.00405	CERN data [18]
p	K	2.52	6.53	Z_{pK} = 0.00212	CERN data [18]
π^+	π^+	15 54	1 495	$Z_{\pi^+\pi^+}$ = 0 11535	FNAL data [19]
π^+	π^-	13 88	3.56	$Z_{\pi^-\pi}$ = 0 04100	FNAL data [19]
π	π^+	13.25	3 75	$Z_{\pi\pi^+}$ = 0.0367	FNAL data [19]
π	π^-	14.46	1 515	$Z_{\pi^+\pi}$ = 0 1134	FNAL data [19]

The p_T integrated Lorentz invariant cross section CERN LEBC EHS data [18] for $\pi^{\pm}$ and $K^{\pm}$ production initiated by pp collisions and FNAL data [19] for $\pi^{\pm}$ production initiated by πp collisions can be fitted by the form (3) whose parametric values are displayed in the Table 2.

The adopted inelastic cross-sections for pp and πp interactions are 35 mb and 22 mb, respectively. The Q-G plasma correction of Z-factors have been made by using the form

$$Z'^{A}_{p\pi^{+}} = 0.79\, Z^{A}_{p\pi^{+}} + 0.21\left[(1-\beta) Z^{A}_{p\pi^{\pm}} + \beta Z^{A}_{p\pi^{-}}\right]. \tag{13}$$

The parametric values of the fitting constants C and n of the relation (3) for hadron production cross sections for $\pi^{\pm}$ and $K^{\pm}$ initiated by pp and πp collisions are given in Table 2.

The Z-factors are corrected for p-air collisions using the methodology of Minorikawa and Mitsui [23] and is shown in the table III. The detail of the procedure is discussed in our earlier work [6]. We have adopted the constant values of σ_{p-air} and $\sigma_{\pi-air}$ cross-sections as 273 mb and 213 mb, respectively from the observations of Boziev *et al.* [30].

Table 3 shows the estimated spectrum weighted moments Z_{hi} for different inclusive reactions $hp \to i^{\pm} X$ for hadron production in air as obtained from CERN [18] for pp interaction and FNAL [19] for $\pi^{\pm}p$ experiments duly corrected for p-air collisions by adopting the procedure [23].

The upper estimates are Z factors corrected for p-air and lower estimates are for A-A collisions estimated from the relation (13) :

Table 3. The estimated Z-factors

Z_{hi}	$Z_{p\pi^+}$	$Z_{p\pi^-}$	Z_{pK^+}	Z_{pK^-}	$Z_{\pi^+\pi^+}$	$Z_{\pi^+\pi^-}$	$Z_{\pi^-\pi^+}$	$Z_{\pi^-\pi^-}$
p-air	0.0451	0.03146	0.00372	0.00216	0.0949	0.0385	0.0348	0.0934
A-A	0.0444	0.0321	0.0036	0.0022	0.0919	0.0412	0.0378	0.0907

Using the relations (7)-(11), Z-factors from Table 3 and the conventional parametric values given in Table 4, the ratio of muon fluxes obtained from kaon decay to that obtained from pion decay has been plotted in Figure 2. At 1 TeV the ratio is about 0.2. The plot indicates that the intensity of muons from kaon decay increases considerably with energy.

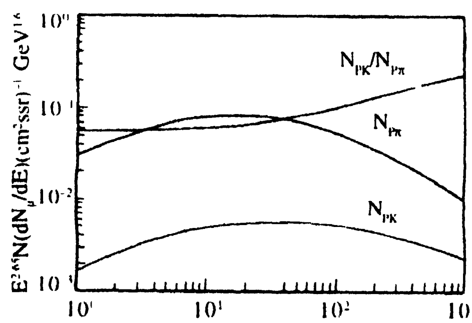

Figure 2. The terms $N_{\mu\pi}$ and $N_{\mu K}$, represent the fluxes of muons obtained from the decay of pions and kaons in the atmosphere derived from the relations (7) and (8), respectively. The term $\frac{N_{\mu K}}{N_{\mu\pi}}$ is the respective ratio of the fluxes of muons

Taking $\lambda_p = 85$ g-cm^{-2}, $\Lambda_p = 110$ g-cm^{-2} the other adopted parametric values are displayed in Table 4.

Table 4. Adopted parametric values

i	λ_i g-cm^2	r_i	H_i GeV	R_i	A_i
π	120	0.78624	118	1.1994	0.7167
K	150	0.52	852	1.3447	0.4994

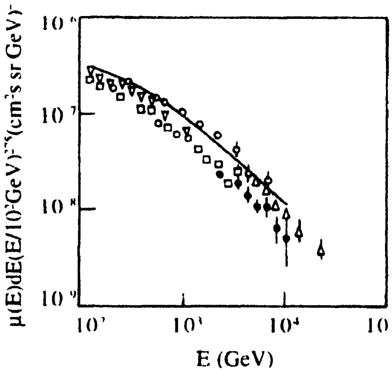

Figure 3. A comparison of the derived muon energy spectrum in the vertical direction with the magnetic spectrograph results ● MUTRON [31], □ DEIS [32], ∇ DESY (Dau *et al* [33]), Δ Moscow University Group [34], o MARS Durham [35]

Full curve (in the Figure 3) is the vertical sea level muon energy spectrum derived from the non-prompt meson decay.

The directly measured magnetic spectrograph data of different authors [30-34] is found comparable to our derived muon energy specturm calculated from the all nucleon spectrum through non-prompt meson production and as a consequence those are treated as the parents responsible for muon production.

In our earlier observation [36] it is found that the contribution of prompt mesons to muon flux is almost negligible below 10 TeV muon energy and for that reason we have neglected their contribution to moderate energy muon flux in the present derivation.

4. Conclusion

The vertical sea level muon energy spectrum has been derived from the latest all primary nucleon spectrum using CERN and FNAL accelerator data on the meson production cross-section fitted to Feynman scaling model and standard meson atmospheric diffusion equation. The derived result is supported by the magnetic spectrograph muon intensity data available from MARS, DEIS, MUTRON and Moscow University experiments. It is also observed that the muon flux obtained from the decay of pions is much higher than that obtained from kaon decay. The ratio of the fluxes of muons from the decay of kaons to muons pions is found to increase with energy and beyond 100 GeV the contribution of kaons to total muon flux is considerable in comparison to that available from pion decay muons. The contribution of prompt muon to muon flux is neglected due their poor contribution to moderately energetic muon fluxes.

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